RESCEU-1/13

A new probe of dark matter properties: gravitational waves from an intermediate mass black hole embedded in a dark matter mini-spike

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An intermediate mass black hole (IMBH) may have a dark matter (DM) mini-halo around it and develop a spiky structure within less than a parsec from the IMBH. When a stellar mass object is captured by the mini-halo, it eventually infalls into such an IMBH due to gravitational wave back reaction which in turn could be observed directly by future space-borne gravitational wave experiments such as eLISA/NGO. In this paper, we show that the GW detectability strongly depends on the radial profile of the DM distribution. So if the GW is detected, the power index, that is, the DM density distribution would be determined very accurately. The DM density distribution obtained would make it clear how the IMBH has evolved from a seed BH and whether the IMBH has experienced major mergers in the past. Unlike the gamma ray observations of DM annihilation, GW is just sensitive to the radial profile of the DM distribution and even to non-interacting DM. Hence the effect we demonstrate here can be used as a new and powerful probe into DM properties.

INTRODUCTION

A large number of astrophysical and cosmological observations provide convincing evidence for the existence of dark matter (DM). The origin and nature of DM remain largely unknown, and are among the most challenging problems in current cosmology and most likely in particle physics.

Recently, the distribution of DM around a black hole (BH) has been under discussion in the context of indirect searches for DM annihilation signals with gamma ray observations. Gondolo and Silk [1] first suggested that the adiabatic growth of a BH creates a high density DM region, called the "spike", which enhances the DM annihilation rate. Subsequent work showed that the existence of a DM spike around a supermassive black hole (SMBH) turns out to be unlikely when one considers the effects of major merger events of the host galaxies [2], off-center formation of the seed BH [3], and scattering of dark matter particles by surrounding stars [4, 5]. On the other hand, a DM "mini-spike" around an intermediate mass black hole (IMBH), with a mass range between 10^2 to $10^6 M_\odot$, may survive if the IMBH never experienced any major mergers [6, 7], as is expected to be the case for the many IMBH that have failed to merge into a supermassive BH.

The existence of such a spike structure is strongly dependent on the details of BH formation and the history of major mergers, which are far from clear. In this Letter, we propose that future gravitational wave (GW) experiments can be used to probe the DM distribution around BHs. The existence of the dense DM region changes the gravitational potential and affects the orbit of an object around the BH. We consider GWs from the coalescence event of a compact binary consisting of a small mass object and an IMBH, and evaluate the modification of the GW signal by the existence of a DM spike associated with the IMBH. Such an event may be observed by future space-based interferometers such as eLISA/NGO [8] and DECIGO [9]. We further discuss whether the eLISA/NGO experiment is sensitive to the modification of the signal by the DM spike.

Note that, while gamma ray observations can find the signal of DM annihilation if DM is a weakly interacting massive particle, the observation of GWs is just sensitive to the gravitational potential of the DM halo and applicable for any type of DM. Therefore, future GW experiments offer a unique opportunity for testing the existence of the DM spike around BHs.

Let us describe the radial profile of the DM spike by a single power-law $\rho \propto r^{-\alpha}$ assuming a spherically symmetric distribution of DM. The adiabatic growth of the BH produces a dense spike in the inner region of the mini-halo within a radius of $r_{\rm sp} \sim 0.2 r_h$, where r_h is the radius of gravitational influence of the BH defined by $M(< r_h) = 4\pi \int_0^{r_h} \rho(r) r^2 dr = 2M_{\rm BH}$, with $M_{\rm BH}$ being the BH mass [4]. The final density profile of the spike depends on the

power-low index $\alpha_{\rm ini}$ of the inner region of the initial mini-halo as, $\alpha = (9-2\alpha_{\rm ini})/(4-\alpha_{\rm ini})$ [1, 12]. If we assume the Navarro, Frenk and White (NFW) profile [10] for the initial condition ($\alpha_{\rm ini}=1$), we get $\alpha = 7/3$. Very steep slope is generically predicted as we find $2.25 < \alpha < 2.5$ for $0 < \alpha_{\rm ini} < 2$.

In summary, in this paper, we assume the DM distribution of a mini-spike is described by

$$\rho(\boldsymbol{x}) = \rho_{\rm sp}\left(\frac{r_{\rm sp}}{r}\right)^{\alpha} \quad (r_{\rm min} \le r \le r_{\rm sp}), \tag{1}$$

where $\rho_{\rm sp}$ is the normalization of the DM density. For an IMBH with the mass of $M_{\rm BH} = 10^3 M_\odot$ and the total mass of the DM mini-halo of $M_{\rm halo} = 10^6 M_\odot$, we get $\rho_{\rm sp} = 379\ M_\odot/{\rm pc}^3$ and $r_{\rm sp} = 0.33{\rm pc}$. Beyond the spike radius $r_{\rm sp}$, the DM distribution obeys the NFW profile with the concentration parameter $c = 6.6$ estimated based on the fitting formula given, e.g., in [11]. The minimum radial distance is taken to be $r_{\rm min} = r_{\rm ISCO}$ where $r_{\rm ISCO}$ is the Innermost Stable Circular Orbit (ISCO) given by $r_{\rm min} = r_{\rm ISCO} = 6GM_{\rm BH}/c^2$. Throughout the paper, we restrict α to below 3, since $r_{\rm sp}$ is nearly the same as $r_{\rm min}$ and the mini-spike virtually vanishes when $\alpha > 3$.

FORMULATION

GWs from binary inspiral

Let us consider gravitational waves from a binary system consisting of an IMBH with a mass of $M_{\rm BH} \sim 10^3 M_\odot$ and a compact object with a mass of $\mu \sim 1 M_\odot$. For simplicity, we make the following idealization. First, we treat the star as a test particle and we call it a "particle" in the following. Second, we assume that the DM density is unperturbed even when the star orbits in the DM halo. Gravitational heating of the DM mini-spike due to the particle may be noticeable about within the Hill sphere of the particle because of the gravity of the central IMBH. In the case of our $1M_\odot$-$10^3 M_\odot$ binary, the Hill radius is 10% of the orbital radius and we ignore possible heating effects in the first order approximation. Then, the equation of motion for the particle is written as

$$\frac{d^2 r}{dt^2} = -\frac{GM_{\rm eff}}{r^2} - \frac{F}{r^{\alpha-1}} + \frac{l^2}{r^3}, \tag{2}$$

where l is the angular momentum of the particle per its mass, and $M_{\rm eff}$ and F is

$$M_{\rm eff} = \begin{cases} M_{\rm BH} - \dfrac{4\pi r_{\rm sp}^{\alpha}\rho_{\rm sp}}{3-\alpha} r_{\rm min}^{3-\alpha} \\ M_{\rm BH} \end{cases}, \quad F = \begin{cases} \dfrac{4\pi G r_{\rm sp}^{\alpha}\rho_{\rm sp}}{3-\alpha} & (r_{\rm min} \le r \le r_{\rm sp}) \\ 0 & (r < r_{\rm min}) \end{cases} \tag{3}$$

In the first term of the right hand side of Eq. (2), the DM mini-halo modifies the effective mass of the central IMBH. The second term contains information of the DM halo radial distribution. The third term represents a centrifugal force. If we assume that the second term is much smaller than the first term,

$$\varepsilon \left(\frac{r}{r_{\rm min}}\right)^{3-\alpha} \ll 1 \qquad \left(\varepsilon \equiv \frac{F r_{\rm min}^{3-\alpha}}{GM_{\rm eff}}\right),$$

we can treat the term which involves information on the DM halo as a perturbation, and expand equations in powers of ε, which is a dimensionless parameter depending the power index α.

When the particle stably orbits around the IMBH at a constant radius R, the left hand side of the equation of motion vanishes. In this case, the GW waveforms are given by

$$h_+ = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\frac{1+\cos^2\iota}{2}\cos(2\omega_s t) \tag{4}$$

$$h_\times = \frac{1}{r}\frac{4G\mu\omega_s^2 R^2}{c^4}\cos\iota\sin(2\omega_s t) \tag{5}$$

to the lowest order approximation where ι is the inclination which is the angle between the normal to the orbit and the line-of-sight, and $2\omega_s$ is the GW frequency.

Waveforms including GW back-reaction

Next, we include the effect of the GW back-reaction within the linearized theory of Einstein's general relativity. The orbital radius and frequency is no longer constant, because GW radiation energy $E_{\rm GW}$ is taken from the rotational energy $E_{\rm orbit}$ of the particle. The relation between the orbital radius R and the time t is given by (e.g., [13]),

$$\frac{dE_{\rm orbit}}{dt} = -\frac{dE_{\rm GW}}{dt}. \tag{6}$$

Using this relation, we can compute the orbital frequency ω_s and R as a function of time. To include the GW back-reaction in the GW waveforms, we replace the constant parameters ω_s and R in Eqs. (4) and (5) by time-dependent functions $\omega_s(t)$ and $R(t)$.

The stationary phase approximation then enables us to obtain the GW waveforms in Fourier space expanded in ε,

$$\tilde{h}_+(f) = \left(\frac{5}{24}\right)^{\frac{1}{2}} \frac{e^{i\Psi(f)}}{\pi^{\frac{2}{3}} f^{\frac{7}{6}}} \frac{c}{r} \left(\frac{GM_c}{c^3}\right)^{\frac{5}{6}} \frac{1+\cos^2\iota}{2} \left[1 + \frac{1}{3}\frac{(2\alpha-23)(\alpha-3)}{7-\alpha} \left(\frac{GM_{\rm eff}}{\pi^2 r_{\rm min}^3 f^2}\right)^{\frac{3-\alpha}{3}} \varepsilon + \cdots\right], \tag{7}$$

$$\tilde{h}_\times(f) = \left(\frac{5}{24}\right)^{\frac{1}{2}} \frac{ie^{i\Psi(f)}}{\pi^{\frac{2}{3}} f^{\frac{7}{6}}} \frac{c}{r} \left(\frac{GM_c}{c^3}\right)^{\frac{5}{6}} \cos\iota \left[1 + \frac{1}{3}\frac{(2\alpha-23)(\alpha-3)}{7-\alpha} \left(\frac{GM_{\rm eff}}{\pi^2 r_{\rm min}^3 f^2}\right)^{\frac{3-\alpha}{3}} \varepsilon + \cdots\right], \tag{8}$$

$$\Psi = 2\pi f\left(t_c + \frac{r}{c}\right) - \Phi_0 - \frac{\pi}{4} + 2\left(\frac{GM_c}{c^3} 8\pi f\right)^{-\frac{5}{3}} + \Delta\Psi, \tag{9}$$

with

$$\Delta\Psi = 2\left(\frac{GM_c}{c^3}8\pi f\right)^{-\frac{5}{3}} \left[\frac{10}{3}\frac{2\alpha-5}{2\alpha-11}\left(\frac{GM_{\rm eff}}{\pi^2 r_{\rm min}^3 f^2}\right)^{\frac{3-\alpha}{3}} \varepsilon - \frac{5}{9}\frac{(2\alpha-1)(4\alpha-11)}{4\alpha-17}\left(\frac{GM_{\rm eff}}{\pi^2 r_{\rm min}^3 f^2}\right)^{\frac{2(3-\alpha)}{3}} \varepsilon^2 + \cdots\right], \tag{10}$$

where t_c is the value of retarded time at coalescence, Φ_0 is the value of the phase at coalescence, $M_c = \mu^{3/5} M_{\rm eff}^{2/5}$ is the chirp mass, and Ψ is the phase of the GW waveform. These expansions are valid for the frequency f for which higher order terms are negligible.

In Eq. (9), the phase of the GW is modified by the presence of the DM, which is expressed in powers of ε. Since GW interferometers are very sensitive to the phase of the signal, this phase difference is crucial for distinguishing the existence of the DM spike. In Fig. 1, we plot the phase difference $\Delta\Psi$ caused by the DM halo, taking into account terms up to second order in ε. We see $\Delta\Psi$ increases for low frequencies and for large α. This can be explained by the fact that the orbit of the object is affected only by the DM mass inside the orbital radius. More phase difference is produced when the inner mass is large. As shown in Fig. 2, the enclosed DM mass increases as the radius or α increases. Since a low frequency of the GW corresponds to a large orbital radius, a large phase difference is produced at low frequencies. Large values of α means a steep density distribution and leads to a large inner mass, which also results in a large phase difference.

OBSERVATION OF GWS

Matched filtering

Let us discuss if this effect is testable by future GW experiments. The search for GW signals is performed by matched filtering analysis, in which one correlates detector output with theoretical template. The signal-to-noise ratio is defined by

$$\left(\frac{S}{N}\right)^2 = 4\frac{\left[\int_{f_{\rm ini}}^{\infty} df\, \frac{\tilde{h}(f)\tilde{h}_t^*(f)}{S(f)}\right]^2}{\int_{f_{\rm ini}}^{\infty} df\, \frac{\left|\tilde{h}_t(f)\right|^2}{S(f)}}, \tag{11}$$

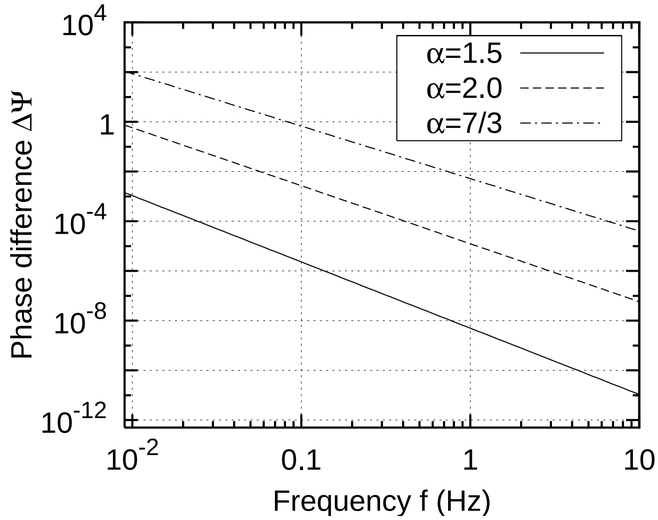

FIG. 1: Phase difference $\Delta\Psi$ against frequency. Solid line is for $\alpha = 1.5$, dashed line is for $\alpha = 2.0$ and dot-dashed line is for $\alpha = 7/3$.

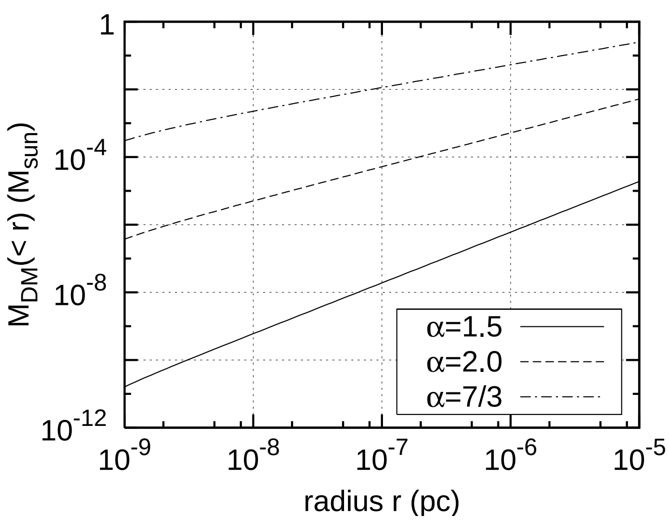

FIG. 2: Mass of DM halo within orbital radius r. Steeper density distributions contain more DM mass within the radius r. The solid line is $\alpha = 1.5$, the dashed line is $\alpha = 2.0$ and the dot-dashed line is $\alpha = 7/3$.

where $\tilde{h}(f)$ is the GW signal coming to the detector, $\tilde{h}_t(f)$ is the template, $S(f)$ is the spectral density of the detector noise, and $f_{\rm ini}$ is the frequency of the inspiral GW when the observation started. In the following example, we assume the eLISA experiment, whose noise spectrum is given in Ref. [8].

In Eq. (11), the numerator is a noise-weighted correlation between the template and the true signal, and the denominator is the renormalization factor. When the template matches the true waveform, S/N is maximized. Thus, S/N is an indicator to tell us whether the waveform of the template is present in the detector or not.

Detectability of the effect of a DM halo around an IMBH

Let us consider an observation of GWs from the $1M_\odot$ particle inspiralling into the $10^3M_\odot$ IMBH, which would be detectable by the eLISA experiment. We assume this binary is surrounded by a DM mini-spike whose distribution

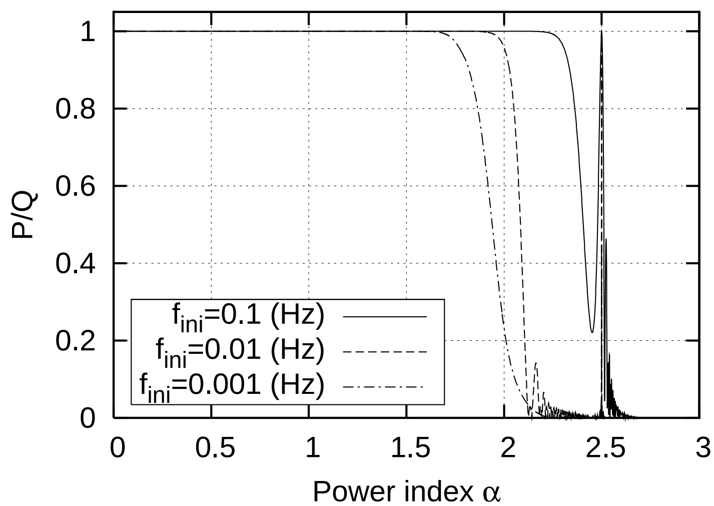

FIG. 3: P/Q against power index α. Three different curves show P/Q for three different values of initial frequency $f_{\rm ini}$, namely differential observation time. The solid line is for $f_{\rm ini} = 0.1$ Hz, the dashed line is for $f_{\rm ini} = 0.01$ Hz and the dot-dashed line is for $f_{\rm ini} = 0.001$ Hz.

is given by Eq. (1). In this setup, a frequency integration from $f_{\rm ini} = 0.01$Hz corresponds to a 4.5 year observation until the coalescence (corresponding roughly to the expected eLISA frequency band and observation time). Note that when $f_{\rm ini} = 0.01$Hz, the particle is about 10^{-8} pc away from the IMBH and well within the mini-spike.

In Fig. 3, we show how much the S/N is degraded when one applies a template predicted without considering the DM effect on the signal with the DM effect. The vertical axis represents a degradation rate P/Q, where P is S/N calculated assuming a template of a waveform without the DM effect ($\varepsilon \to 0$) and Q is S/N calculated with a template including the DM effect up to the second order. If the effect of the DM is small, there is little difference between the two templates and P/Q becomes 1. Conversely, if DM potential induces significant phase difference, the value of P decreases, since the template and the signal have less correlation.

As discussed in the previous section, the phase difference becomes significant for large α, and, from Fig. 3, we find P/Q largely deviates from 1 for $\alpha \gtrsim 2$. This result indicates that GW observation can distinguish whether a DM halo of $\alpha \gtrsim 2$ exists around the IMBH. In order to extract inspiral signals under the effect of a DM halo, we must prepare templates including the DM effect. In Fig. 3, we also plot the cases for different initial integration frequencies, which corresponds to different observation time. Since the phase difference becomes larger at low frequencies, P/Q is suppressed for smaller value of α when one observes a longer time period. The peak seen at $\alpha = 2.5$ originates from the zero crossing of the first term of Eq. (10).

CONCLUSION

We have demonstrated a method to probe the DM distribution around an IMBH by using GW direct detection experiments. Considering a GW inspiral signal from inspiral of a compact object falling in an IMBH, we have computed how the GW waveform is modified by the gravitational potential of the DM halo. Thanks to the fact that a GW interferometer measures the phase of the signal with very good accuracy, we found that a GW experiment such as eLISA/NGO is sensitive to the phase shift caused by the DM potential. Accordingly, GW observation may provide a new observational probe for the DM distribution near BHs and would be helpful for testing the existence of a dense DM minihalo, the so-called DM spike. This may even offer hints to the formation history of BHs, since formation of DM spikes strongly depends on how BHs evolved.

In future work, we plan to extend the investigation for different values of mass distribution parameters, such as the power index α and the mass of the compact objects and the halo. We will also estimate to what degree future GW experiments can determine the mass distribution by computing expected errors on the parameters.

Finally we remark on the dynamical friction effect. In this paper, we have ignored dynamical friction. Even if GW back reaction is ignored, the dynamical friction alone makes the smaller compact object infall into the IMBH. Using

the Chandrasekhar formula (e.g., [14]), we found however that this effect produces negligible phase difference when computing the SN degradation rate.

We thank Jun'ichi Yokoyama for his valuable comments. This work is supported by the Grant-in-Aid for Scientific Research (B) No. 23340058 from the Japan Society for the Promotion of Science. SK is partially supported by the Grant-in-Aid for Scientific Research No. 24740149. The work of JS is supported in part by ERC project 267117 (DARK) hosted by Université Pierre et Marie Curie - Paris 6.

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